European Workshop on Computational Geometry

EuroCG 2015

March 15-18, Ljubljana, Slovenia

http://eurocg15.fri.uni-lj.si
LPP bus station
"Živalski vrt ZOO"

Building X

Main entrance
Ground floor

Restaurant
2nd floor

LPP bus station
"Viško polje"

Faculty building
1st floor

P19

P21

Coffee break

P01
Welcome to EuroCG 2015!

Here is some basic information about the 31st European Workshop on Computational Geometry (EuroCG 2015), hosted by University of Ljubljana, Faculty of Computer and Information Science. The EuroCG is an annual informal workshop, whose goal is to provide a forum for scientists to meet, present their work, interact, and establish collaborations, in order to promote research in the field of Computational Geometry. The workshop aims at providing an informal atmosphere, where established and young researchers have a possibility for a productive exchange of ideas and collaboration.

To help you decide which talk to attend, we have collected short abstracts of each paper. Because of the constraints given by EasyChair, the abstracts are those given at EasyChair at the time of submission, unless we were notified of changes in the title. We also have included the abstract of the 3 invited talks, delivered by Aleš Leonardis, Kurt Mehlhorn, and Bojan Mohar.

Participants, speakers, and organizers are the visible part of the event, but we should be grateful to all the authors and the PC members for all their work. We gratefully thank the European Science Foundation (ESF), under the EUROCORES Programme EuroGIGA, for their financial support that lead to a substantial reduction in the registration fees. Finally, we would like to thank again the members of the Organizing Committee for their work to make the event as smooth as possible.

We wish you an enjoyable and successful experience, and hope to see you again in future editions!

Andrej Brodnik and Sergio Cabello
Co-chairs of the Organization Committee EuroCG 2015
Dear participants of the EuroCG 2015, dear colleagues,

The Faculty of Computer and Information Science at the University of Ljubljana is the leading institution in the field of computer and information science in Slovenia. Since its first study programme in Computer Science in 1973, it has had a lengthy roster of alumni, many of whom have achieved distinction in academic and professional careers in Slovenia and abroad.

In summer 2014 we moved to new premises. The new Faculty building, which was funded by EU, gives us excellent working conditions that allow us to prosper in our endeavours. We have set ambitious goals in terms of international networking and cooperation therefore we are honoured to host the 31st EuroCG. Computational geometry is an important branch of Computer Science and it has always been an integral part of our research and study process at the Faculty. Many of our graduates and academics are renowned internationally in the field and this year they have taken up the challenge of organizing the annual gathering of Computational geometry community. We take very seriously the responsibility of providing the best conditions for your work since it is the cooperation across all borders that truly can make a progress in any scientific endeavour.

In recent years we have been expanding research competences of our faculty to fit a wider spectrum of promising areas by attracting expe-
rienced researchers and teachers. In 2014 we hired new foreign teach-
ers with internationally recognized research record and ability to di-
rect research of highest quality. Furthermore, we are intensifying co-
operation with related institutions in neighbouring and other coun-
tries. We implemented the double master degree program in Com-
puter Science with Graz University of Technology and we are working
towards establishment of further double degree studies.
One of our prime goals is also to make our studies more accessible to
international students. Part of the Master’s Studies and the entire
Doctoral Programme are conducted in English and particular atten-
tion is given to attracting promising international doctoral students.
In addition to our core programme of Computer Science we foster an
interdisciplinary approach through study programmes with selected
faculties of the University of Ljubljana and other European universi-
ties, such as the joint degree of Middle European Interdisciplinary
Master Programme in Cognitive Science.
This Faculty is therefore the perfect environment for the informal at-
mosphere of EuroCG workshops. With many years of experience and
a new momentum the faculty staff is always welcome to assist and
the vibrant student activity at the beginning of the spring semester
will contribute some additional positive energy.
I hope you will find excellent presentations, inspiring debates and
fresh ideas during the EuroCG conference. I am sure it will establish
new connections amongst scientists in computer science community
and further deepen the cooperation of research institutions across Eu-
rope and wider.

Nikolaj Zimic
Dean
Faculty of Computer and Information Science
General Information

Ljubljana airport – city connections

Recommended way of getting from the airport to Ljubljana city is using GRH chaffeur services. They offer transports according to the aircraft arrivals/departures 24/7 and they will drive you all the way to your hotel. Trip takes around 30 minutes. One way prices range from 12€ per person for a Shuttle bus (max. waiting time 1h) to 40€ per car for a private transfer (1-3 persons, no waiting time). Register your transfer at http://grh.si and select the conference “EUROCG 2015”. In case of any trouble contact GRH directly on +386 41 376 964.

Public buses are also operating every hour on working days and every two hours on weekends. Bus takes around 55 minutes and costs 4.10€ one way. Please note that the bus arrives to Ljubljana central bus station which is not in a walkable distance to the conference venue. For information on schedules check http://ap-ljubljana.si/eng. Airport is called “Letališče Brnik” in Slovenian and the central station is called “Ljubljana AVTOBUSNA POSTAJA” or “AP Ljubljana” in short.

Public transportation in Ljubljana

EuroCG 2015 is held in the new building of the Faculty of Computer and Information Science. The postal address is Večna pot 113 and the nearest LPP bus stations are called “Živalski vrt ZOO” (bus lines 18 and 18L) and “Viško polje” (bus lines 14 and 14B). A detailed map of LPP bus lines is located on page 2 of this booklet. In order to use LPP buses, one must buy the Urbana single city card, available at 39 Urbanomats across the city near the bus stations. Participants staying at Four Points by Sheraton Ljubljana Mons have an organized transportation provided by the hotel from the hotel to the faculty and back each morning and afternoon.
Information desk, registration

On Sunday, the conference registration desk will be located at the atrium, where the reception will take place from 17:30 to 20:00. From Monday to Wednesday the conference information and the registration desk will be located in the yellow facility in the main hall of the faculty from 8:00 to 17:00. Moreover, our students and other staff are around and are more than happy to answer your questions.

Internet access

There is a freely accessible 802.11n WLAN available. Participants can use either the Eduroam account, if they have one, or connect to EU-ROCG wireless network and enter password freefri2015.

Technical support

EuroCG 2015 employs technical staff in all lecture rooms. They wear a badge with a red rectangle and an “Organizer” or “Staff” label. If you encounter technical problems, do not hesitate to contact any of them.

Session organization

EuroCG 2015 features workshops in two parallel sessions. In order to allow attendees to easily switch sessions between talks, we synchronize all the lecture rooms with a Conference Session Syncronizer (CoSeSy). Both rooms are equipped with a big screen that indicates the remaining time for each talk. All slots are 20 minutes long and speakers are requested to observe the time: 15 minutes for the talk (green color+orange at the end of the talk), 2 minutes for questions (red) and 3 minutes for breaks between talks for changing the speaker and set up equipment (blue). CoSeSy timers can be accessed by anyone by visiting the following URLs ranging from Monday to
Wednesday:

Each speaker will have a laser pointing device, presentation computer with a display and a video projector available. Presentation computers run Windows 8.1 and have Microsoft Office 2013 Professional, LibreOffice 4.4.1, Acrobat reader XI, GSView v5.0 with Ghostscript 9.15 and VLC media player installed. All speakers should report to the session chair before the session starts.

The main responsibility of the session chair is to keep the session running on schedule as indicated by the provided timer. Session chairs should arrive early and before the start of the session invite speakers to upload their presentations to the presentation computer. If a speaker needs to use her/his own equipment, (s)he should test it before the session starts. Most importantly, please ensure that speakers start and stop on time. If talk ends early, we would kindly ask you to wait until the break between talks ends and the timer turns green again. In case of any technical issues, there will be a member of staff available in each lecture room.

Catering

Lunches are organized in faculty's restaurant on the 2nd floor of Building X. Coffee breaks and snacks are located in a separated part of the faculty's main hall. Lunches are organized as a self-serviced standing buffets. All EuroCG attendees are required to show their badge when entering the restaurant.

Welcome reception and the conference dinner

Welcome reception with the welcome drink will be organised in downtown Ljubljana in the atrium of the Research Centre of the Slovenian
Academy of Sciences and Arts on Sunday at 18:00 (https://goo.gl/maps/z3XCE).
On Tuesday evening there will be a conference dinner at hotel Four Points by Sheraton Ljubljana Mons (https://goo.gl/maps/EQwYF). Number 14 in a blue circle marks the place. There will be organized a transport from downtown and back.

Rough schedule:
18:30 – Bus leaving downtown from the bus station “Tavčarjeva”
18:35 – Bus leaving faculty's bus station “Živalski vrt ZOO”
19:00 – Conference dinner
21:30 – Departure back

Notice bus stations “Tavčarjeva” and “Živalski vrt ZOO” colored red on the map on page 2 of this booklet.

Organizing Committee

EuroCG 2015 is organized by the University of Ljubljana, Faculty of Computer and Information Science and the Faculty of Mathematics and Physics.

Organizing committee:

Andrej Brodnik
Sergio Cabello

Gašper Fele-Žorž
Matevž Jekovec

Jurij Mihelič
Uroš Čibej
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Dear reader,

here are the abstracts of works presented at the 31st European Workshop on Computational Geometry (EuroCG 2015), held on March 15-18, 2015 in Ljubljana, Slovenia. The event was hosted by the University of Ljubljana, Faculty of Computer and Information Science.

The EuroCG is an annual, informal workshop whose goal is to provide a forum for scientists to meet, present their work, interact, and establish collaborations, in order to promote research in the field of Computational Geometry. The workshop aims at providing an informal atmosphere where established and young researchers have a possibility for a productive exchange of ideas and collaboration.

EuroCG does not have formally reviewed proceedings, although the contributions are reviewed and certain improvements to authors are suggested by the Programme Committee. This volume contains a collection of 64 abstracts of talks presented at the workshop. The abstracts should be regarded as preprints, and therefore results presented at EuroCG are often also submitted to peer-reviewed conferences and journals.

There were 77 submissions made to EuroCG 2015, and two of them were not considered because of their format. Each remaining submission was reviewed by at least two members of the Programme Committee. Most of the work of the Program Committee was done through EasyChair. The Program Committee finally decided to accept 66 papers for presentation. Since authors of two submissions were not able to present their papers the final number of papers was 64.

Besides the contributing talks, we also had 3 invited talks, delivered by Aleš Leonardis, Kurt Mehlhorn, and Bojan Mohar. A very short description is also included in this volume.

Such a meeting requires the effort of a lot of parties. We would like to thank the authors for submitting their abstracts and the members
of the Program Committee for their work on selecting the papers.
Last, we thank EasyChair for making its valuable platform available for free.

*Andrej Brodnik and Segio Cabello*

*EuroCG 2015 Program Chairs*

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Gerth Stølting Brodal
Andrej Brodnik
Sergio Cabello
Paz Carmi
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Hierarchical Compositional Representations of Structure for Computer Vision and Robotics

Aleš Leonardis, University of Birmingham, School of Computer Science

Modeling, learning, recognizing, and categorizing visual entities has been an area of intensive research in the vision and robotics communities for several decades. While successful partial solutions tailored for particular tasks and specific scenarios have appeared in recent years, more general solutions, which would be applicable to a variety of different tasks and would scale favorably with a large number of visual entities, are yet to be developed. Ultimately, the goal is to design and implement proper structures and mechanisms that would enable efficient learning, inference, and, when necessary, augmentation and modifications of the acquired visual knowledge in general scenarios. Recently, it has become increasingly clear that new approaches are needed to tackle these problems and there have been several indications that possible solutions should be sought in the framework of hierarchical architectures. Among various design choices related to hierarchies, compositional hierarchies show a great promise in terms of scalability, real-time performance, efficient structured online learning, shareability, and knowledge transfer. In this talk I will first present our work on compositional hierarchies related to visual representations of 2D and 3D object shapes and then conclude with some ideas towards generalizing the proposed approach to other visual entities and modalities.

Immersions of graphs and digraphs

Bojan Mohar, Simon Fraser University

A graph $G$ contains another graph $H$ as an *immersion* if there is an
injective mapping $\iota : V(H) \rightarrow V(G)$ and for each edge $uv \in E(H)$ there is a path $P_{uv}$ in $G$ joining vertices $\iota(u)$ and $\iota(v)$ such that the paths $P_{uv}$ ($uv \in E(H)$) are pairwise edge-disjoint. If the paths are internally disjoint from $\iota(V(H))$, then we speak of a strong immersion. One can define (strong) immersions of digraphs in the same way.

Nash-Williams conjectured that graphs are well-quasi ordered for the relation of immersion containment. The conjecture was proved by Robertson and Seymour (Graph minors XXIII. Nash-Williams' immersion conjecture, J. Combinatorial Theory, Ser. B 100 (2010), 181--205) for weak immersions.

Recent interest in graph and digraph immersions resulted in a variety of new discoveries. The speaker will enlighten some of these achievements.

**Computing Real Roots of Real Polynomials and its Application in Computational Geometry**

Kurt Mehlhorn, Max-Planck-Institut für Informatik

I also discuss recent advances in the computation of real roots of real polynomials. Near optimal solutions for the more general problem of isolating the complex roots of complex polynomials are known for quite some time (V. Pan, 2002). The new algorithms achieve the same time complexity for a sub-problem and are considerably simpler. I also discuss application to computational geometry, in particular, cylindrical algebraic decomposition. The talk is based on joint work with Michael Sagraloff.

**Session 1A**

*A linear-time algorithm for the geodesic center of a simple polygon*

Hee-Kap Ahn, Luis Barba, Prosenjit Bose, Jean Lou de Carufel, Matias Korman and Eunjin Oh

Given two points in a simple polygon $P$ of $n$ vertices, its geodesic dis-
tance is the length of the shortest path that connects them among all paths that stay within \( P \). The geodesic center of \( P \) is the unique point in \( P \) that minimizes the largest geodesic distance to all other points of \( P \). In 1989, Pollack, Sharir and Rote [Disc. & Comput. Geom. 89] showed an \( O(n \log n) \)-time algorithm that computes the geodesic center of \( P \). Since then, a longstanding question has been whether this running time can be improved (explicitly posed by Mitchell [Handbook of Computational Geometry, 2000]).

In this paper we affirmatively answer this question and present a linear time algorithm to solve this problem.

**Computing the s-kernel of Orthogonal Polygons**

Leonidas Palios

Two points \( p, q \) of an orthogonal polygon \( P \) are s-visible from one another if there exists a staircase path (i.e., an \( x- \) and \( y- \) monotone chain of horizontal and vertical line segments) from \( p \) to \( q \) that lies in \( P \). The s-kernel of \( P \) is the set of points of \( P \) from which all points of \( P \) are s-visible. Note that the s-kernel of a polygon may be empty.

We are interested in the problem of computing the s-kernel of a given orthogonal polygon. The problem has been considered by Gewali [1] who described an \( O(n) \)-time algorithm for simple orthogonal polygons on \( n \) vertices and an \( O(n^2) \)-time algorithm for orthogonal polygons with holes; the latter is worst-case optimal since the s-kernel of an orthogonal polygon with holes may have \( \Omega(n^2) \) size.

In this paper, we give a simple output-sensitive algorithm for the problem. For an \( n \)-vertex orthogonal polygon \( P \) that has \( h \) holes, our algorithm runs in \( O(n + h \log h + k) \) time where \( k = O(h^2) \) is the number of connected components of the s-kernel of \( P \). Additionally, a modified version of our algorithm enables us to compute the number \( k \) of connected components of the s-kernel in \( O(n + h \log h) \) time.
Optimizing an oriented convex hull with two directions
Carlos Alegria Galicia, David Orden, Carlos Seara and Jorge Urrutia
Given a set $P$ of $n$ points in the plane in general position, we generalize the rectilinear convex hull $\text{RH}(P)$ to the case of the two lines forming an angle $\beta \in [0, \pi]$. We show:
(i) How this hull $O^2\mathcal{H}_\beta(P)$ can be computed and maintained while $\beta$ changes in $[0,\pi]$.
(ii) How to determine the angle $\beta$ for which $O^2\mathcal{H}_\beta(P)$ has optimal area or perimeter.
Our algorithm runs in optimal $\Theta(n \log n)$ time and $O(n)$ space.

Session 1B

A lower bound on opaque sets
Akitoshi Kawamura, Sonoko Moriyama, Yota Otachi and Janos Pach
It is proved that the total length of any set of countably many rectifiable curves, whose union meets all straight lines that intersect the unit square $U$, is at least 2.00002. This is the first improvement on the lower bound of 2 established by Jones in 1964. A similar bound is proved for all convex sets $U$ other than a triangle.

Approximating the Colorful Caratheodory Theorem
Wolfgang Mulzer and Yannik Stein
Let $P_1, \ldots, P_{d+1} \subset \mathbb{R}^d$ be $d$-dimensional point sets such that the convex hull of each $P_i$ contains the origin. We call the sets $P_i$ color classes, and we think of the points in $P_i$ as having color $i$. A colorful choice is a set with at most one point of each color. The colorful Caratheodory theorem guarantees the existence of a colorful choice whose convex hull contains the origin. So far, the computational complexity of finding such a colorful choice is unknown.
An $m$-colorful choice is a set that contains at most $m$ points from each color class. We present an approximation algorithm that computes for any constant $\varepsilon > 0$, an $\lceil \varepsilon(d+1) \rceil$-colorful choice contain-
ing the origin in its convex hull in polynomial time. This notion of approximation has not been studied before, and it is motivated through the applications of the colorful Caratheodory theorem in the literature. Second, we show that the exact problem can be solved in $d^{O(\log d)}$ time if we have $O(d^2 \log d)$ color classes. This improves the trivial $d^{O(d)}$ algorithm asymptotically.

**Packing Segments in a Simple Polygon is APX-hard**

Heuna Kim and Tillmann Miltzow

For a given set of line segments and a polygon $P$ in the plane, we study the problem to find the maximum number of segments that can be disjointly embedded by translation into $P$. We show APX-hardness of this problem and discuss variations. This problem can be considered in two respects: as a variant of the Kakeya problem and as a maximum-packing problem for line segments.

**Session 2A**

**Finding Pairwise Intersections Inside a Query Rectangle**

Mark de Berg and Ali D. Mehrabi

We study the following problem: preprocess a set $S$ of objects in the plane into a data structure that allows us to efficiently report all pairs of objects from $S$ that intersect inside an axis-aligned rectangular query range $Q$. We present data structures of size $O(n \text{ polylog } n)$ and with query time $O((k+1)\text{polylog } n)$ time, where $k$ is the number of reported pairs, for two classes of objects: axis-aligned rectangles and objects with small union complexity.

**Computing the Smallest Color-Spanning Equaliteral Triangle**

Javad Hasheminejad, Payam Khanteimouri and Ali Mohades

Let $P$ be a set of $n$ colored points with $k$ colors in the plane. A region is color-spanning if it contains at least one point from each color. In this paper, we study the problem of computing the smallest color-spanning equilateral triangle whose one side is parallel to $x$-axis.
We first show that the number of the minimal color-spanning equilateral triangles is $O(n)$ in the worst case. Then, we present an efficient algorithm running in $O(n \log n)$ time to solve the problem. Finally, we show that our algorithm can be used to compute a 2-approximation of the smallest perimeter color-spanning convex hull.

**Elastic Shape Matching for Translations under the Manhattan Norm**  
**Fabian Stehn, Christian Knauer and Luise Sommer**

The Elastic shape matching (ESM) framework is a generalization of the well-studied geometric shape matching problems. For a geometric shape matching problem, one seeks a single transformation (drawn from an appropriate class of transformations) that if applied to a geometric object (the pattern) minimizes the distance of the transformed object to another geometric object (the model).

In an ESM problem, the pattern is partitioned into parts which are transformed by a transformation ensemble (a collection of transformations) to minimize the distance of the individually transformed parts to the model under the constraint, that some transformations of the ensemble have to be similar.

We present algorithms to solve decision variants of ESM problems under translations for point sets under Hausdorff distance (with respect to the Manhattan metric and other polygonal metrics), if the dependencies of the transformations that are forced to be similar form a tree.

**Session 2B**

**Combinatorics of edge 2-transmitter art gallery problems**  
**Sarah Cannon, Thomas Fai, Justin Iwerks, Undine Leopold and Christiane Schmidt**

We give sufficiency and necessity results for edge 2-transmitters in general, monotone, orthogonal and monotone, and orthogonal polygons.
Chromatic Guarding of Orthogonal Polygons with Orthogonal Visibility
Frank Hoffmann, Klaus Kriegel, Subhash Suri, Kevin Verbeek and Max Willert

We address both the strong and the conflict-free chromatic version of the classic Art Gallery Problem. Assume a simple orthogonal polygon $P$ is guarded by a finite set of point guards and each guard is assigned one of $t$ colors. Such a chromatic guarding is said to be conflict-free if each point $p \in P$ sees at least one guard with a unique color among all guards visible from $p$. The guarding is strong if all guards visible from a point have different colors. The goal is to establish bounds on the numbers $\chi_{cf}(n)$ and $\chi_{st}(n)$ of colors sufficient to guarantee the existence of a conflict-free, respectively strong chromatic guarding for any $n$-vertex polygon. In this paper, we assume the $r$-visibility model instead of standard line visibility. Points $p$ and $q$ in an orthogonal polygon are $r$-visible to each other if the rectangle spanned by the points is contained in $P$. For this model we show tight bounds on the number of colors: $\Theta(\log \log n)$ for conflict-free and $\Theta(\log n)$ for strong guarding. Our results can be interpreted as coloring results for special geometric hypergraphs.

Special Guards in Chromatic Art Gallery
Hamid Hoorfar and Ali Mohades

We present two new versions of the chromatic art gallery problem that can improve upper bound of the required colors pretty well. In our version, we employ restricted angle guards so that these modern guards can visit $\alpha$-degree of their surroundings. If $\alpha$ is between 0 and 180 degree, we demonstrate that the strong chromatic guarding number is constant. Then we use orthogonal 90-degree guards for guarding the polygons. We prove that the strong chromatic guarding number with orthogonal guards is the logarithmic order. First, we show that for the special cases of the orthogonal polygon such as snake polygon, staircase polygon and mounts polygon, the number of colors...
is constant. We decompose the polygon into parts so that the number of the conflicted parts is logarithmic and every part is snake. Next, we explain the chromatic art gallery for the orthogonal polygon with guards that have rectangular visibility. We prove that the strong chromatic guarding number for orthogonal polygon with rectangular guards is logarithmic order, too. We use a partitioning for orthogonal polygon such that every part is a mount, then we show that a tight bound for strong chromatic number with rectangular guards is $\theta(\log n)$.

Session 3A

Column Planarity and Partial Simultaneous Geometric Embedding for Outerplanar Graphs

Luis Barba, Michael Hoffmann and Vincent Kusters

Given a graph $G=(V,E)$, a set $R \subseteq V$ is column planar in $G$ if we can assign $x$-coordinates to the vertices in $R$ such that any assignment of $y$-coordinates to $R$ gives a partial embedding of $G$ that can be completed to a plane straight-line embedding of the whole graph. This notion is strongly related to unlabeled level planarity. We prove that every outerplanar graph on $n$ vertices contains a column planar set of size at least $n/2$.

We use this result to show that every two outerplanar graphs $G_1$ and $G_2$ on the same set $V$ of $n$ vertices admit an $(n/4)$-partial simultaneous geometric embedding (PSGE): a plane straight-line drawing of $G_1$ and a plane straight-line embedding of $G_2$ such that $n/4$ vertices are mapped to the same point in the two drawings. This is a relaxation of the well-studied notion of simultaneous geometric embedding, which is equivalent to $n$-PSGE.
All Good Drawings of Small Complete Graphs
Bernardo Ábrego, Oswin Aichholzer, Silvia Fernández-Merchant, Thomas Hackl, Jürgen Pammer, Alexander Pilz, Pedro Ramos, Gelasio Salazar and Birgit Vogtenhuber

Good drawings, also known as simple topological graphs, have recently attracted attention as generalizations of geometric graphs, in connection with the crossing number, and as data structures in their own right. We are in particular interested in good drawings of the complete graph. In this extended abstract, we describe our techniques for generating all different weak isomorphism classes of good drawings of the complete graph for up to nine vertices. In addition, all isomorphism classes were enumerated. As an application of the obtained data, we present several existential and extremal properties of these drawings.

A Linear-Time Algorithm for the Queue-Numbers of Proper Triangulated Cacti
Toru Hasunuma

A proper triangulated cactus is a graph in which every block is a maximal outerplanar graph and there is no leaf. We present a linear-time algorithm for computing the queue-numbers of proper triangulated cacti.

Session 3B

Simple strategies versus optimal schedules in multi-agent patrolling
Akitoshi Kawamura and Makoto Soejima

Suppose that we want to patrol a fence (line segment) using k mobile agents with given speeds $v_1, \ldots, v_k$ so that every point on the fence is visited by an agent at least once in every unit time period. A simple strategy where the i-th agent moves back and forth in a segment of length $v_i/2$ patrols the length $(v_1 + \cdots + v_k)/2$, but it has been shown recently that this is not always optimal. Thus a natural
question is to determine the smallest $c$ such that a fence of length $c(v_1 + \cdots + v_k)/2$ cannot be patrolled. We give an example showing $c \geq 4/3$ (and conjecture that this is the best possible). We also consider a variant of this problem where we want to patrol a circle and the agents can move only clockwise. We can patrol a circle of perimeter $r v_r$ by a simple strategy where the $r$ fastest agents move at the same speed. We give an example where we can achieve the perimeter of $1.05 \max_r rv_r$ (and conjecture that this constant can be arbitrary big). We propose another variant where we want to patrol a single point under the constraint that each agent $i = 1, \ldots, k$ can visit the point only at a predefined interval of $a_i$ or longer. This problem can be reduced to the discretized version where the $a_i$ are integers and the goal is to visit the point at every integer time. It is easy to see that this discretized patrolling is impossible if $1/a_1 + \cdots + 1/a_k < 1$, and that there is a simple strategy if $1/a_1 + \cdots + 1/a_k \geq 2$. Thus we are interested in the smallest $c$ such that patrolling is always possible if $1/a_1 + \cdots + 1/a_k \geq c$. We prove that $\alpha \leq c < 1.546$, where $\alpha = 1.264\ldots$ (and conjecture that $c = \alpha$). We also discuss the computational complexity of related problems.

**Continuous Geometric Algorithms for Robot Swarms with Multiple Leaders**

Maximilian Ernestus, Sándor Fekete, Michael Hemmer and Dominik Krupke

We consider the problem of building a dynamic and robust network between mobile base stations with the help of a large swarm of robots in the continuous Euclidean plane. Individually, the robots have limited capabilities, both in terms of global information and computation. We propose a set of local continuous algorithms that in combination produce a generalization of a Euclidean Steiner tree. At any stage, the resulting overall shape aims achieves a good compromise
between local thickness, global connectivity, and flexibility to further continuous motion of the base stations.

**Randomized Strategy for Walking in Streets for a Simple Robot**  
**Azadeh Tabatabaei and Mohammad Ghodsi**

We consider the problem of walking in an unknown street, for a robot that has a minimal sensing capability. The robot is equipped with a sensor that only detects the discontinuities in depth information (gaps) and can locate the target point as enters in its visibility region. We propose a randomized search strategy that generates a search path for the simple robot to reach the target, starting from s. Even with such a limited capability, we prove that the expected distance traveled by the robot is at most a constant times longer than the shortest path to reach the target.

**Session 4A**

**Orienting triangulations**  
**Boris Albar, Daniel Gonçalves and Kolja Knauer**

We prove that any triangulation of a surface different from the sphere and the projective plane admits an orientation without sinks such that every vertex has outdegree divisible by three. This confirms a conjecture of Barat and Thomassen and is a step towards a generalization of Schnyder woods to higher genus surfaces.

**Lattice 3-polytopes with six lattice points**  
**Monica Blanco and Francisco Santos**

We completely enumerate lattice 3-polytopes of width larger than one and with exactly 6 lattice points: There are 74 of width 2, two of width 3, and none of larger width.

According to the number of interior points, these 76 polytopes divide into 23 tetrahedra with two interior points, 49 polytopes with one interior point and only 4 hollow polytopes (two tetrahedra, one quadrangular pyramid and one triangular bipyramid).
**Automatic Proofs for Formulae Enumerating Proper Polycubes**

Gill Barequet and Mira Shalah

In this paper we develop a general framework for computing formulae enumerating polycubes of size \( n \) which are proper in \( n-k \) dimensions (i.e., spanning all \( n-k \) dimensions), for a fixed value of \( k \). (Such formulae are central in the literature of statistical physics in the study of percolation processes and collapse of branched polymers.) We reaffirm the already-proven formulae for \( k \leq 3 \), and prove rigorously, for the first time, that the number of polycubes of size \( n \) that are proper in \( n-4 \) dimensions is \( 2^{n-7}n^{n-9}(n-4)(8n^8 - 128n^7 + 828n^6 - 2930n^5 + 7404n^4 - 17523n^3 + 41527n^2 - 114302n + 204960)/6 \).

**Compact families of Jordan curves and convex hulls in three dimensions**

Colm O Dunlaing

We prove that for certain families of semi-algebraic convex bodies in \( \mathbb{R}^3 \), the convex hull of \( n \) disjoint bodies has \( \mathcal{O}(n\lambda_s(tn)) \) features, where \( s \) and \( t \) are constants depending on the family: \( \lambda_s(n) \) is the maximum length of order-\( s \) Davenport-Schinzel sequences with \( n \) letters. The argument is based on an apparently new idea of \'compact family\' of convex bodies or discs, and of \'crossing content\' among disc intersections.

**Session 4B**

**New Geometric Algorithms for Staged Self-Assembly**

Erik D. Demaine, Sándor Fekete and Arne Schmidt

We consider staged self-assembly, in which square-shaped Wang tiles can be added to bins in several stages. Within these bins the tiles may connect to each other, depending on the glue types of their edges. In general, self-assembly constructs complex (polyomino-shaped) structures from a limited set of square tiles. Previous work by Demaine et al. considered a setting in which assembly proceeds in stages. It was shown that a relatively small number of tile types suf-
fices to produce arbitrary shapes; however, these constructions were only based on a spanning tree of the geometric shape, so they did not produce full connectivity of the underlying grid graph. We present new systems systems for stages assembly to assemble a fully connected polynomino in $O(\log^2(n))$. Our construction works even for shapes with holes and uses only a constant number of glues and tiles.

**Caging polygons by a Finger and a Wall**

Bahareh Banyassady, Mansoor Davoodi and Ali Mohades

The finger-wall caging grasps of polygons is studied in this paper. An object is caged by a finger-wall grasp if it is impossible for the object to move to arbitrary placement far from its initial placement without penetrating the finger or the wall. We present an algorithm in $O(n^2 \log n)$ time for computing all configurations of the finger-wall grasp which cage a given polygon with $n$ edges. In addition, the output set of all caging configurations can be queried in $O(\log n)$ time to check whether a given arbitrary finger-wall grasp cage the polygon.

**Subquadratic Medial-Axis Approximation for smooth Curves in $\mathbb{R}^3$**

Christian Scheffer

We present the first algorithm to approximate the medial axis of a smooth, closed curve $\gamma \subset \mathbb{R}^3$ in subquadratic time. Our algorithm works on a sufficiently dense $\varepsilon$-sample and comes with a convergence guaranty for the non-discrete, but continuous approximation object.

**Adaptive analysis-suitable T-mesh refinement with linear complexity**

Philipp Morgenstern and Daniel Peterseim

We present an efficient adaptive refinement procedure that preserves analysis-suitability of the T-mesh, this is, the linear independence of the T-spline blending functions. We prove analysis-suitability of the overlays and boundedness of their cardinalities, nestedness of the generated T-spline spaces, and linear computational complexity of the refinement procedure in terms of the number of marked and generated mesh elements.
**Session 5A**

*The Slope Number of Segment Intersection Graphs*

*Udo Hoffmann*

We proof that it is NP-hard to determine the minimal number of slopes that is required to draw a segment representation of a segment intersection graph. As a side product we obtain new proofs for the NP-hardness of the recognition of grid, segment and pseudosegment graphs. We show as well, that the minimum number of slopes of a segment graph can drop arbitrarily upon the removal of a single vertex.

*Recognizing Weighted Disk Contact Graphs*

*Boris Klemz, Martin Nöllenburg and Roman Prutkin*

Disk contact representations realize graphs by mapping vertices to interior-disjoint disks in the plane such that disks touch each other if and only if the corresponding vertices are adjacent. Deciding whether a vertex-weighted graph can be realized such that the disks' radii coincide with the vertex weights has been proven NP-hard. In this work, we analyze the problem for special graph classes and show that it remains hard even for very basic ones, thereby strengthening previous NP-hardness results. On the positive side, we present linear-time algorithms for two restricted versions of the problem.

*The Complexity of the Partial Order Dimension Problem – Closing the Gap*

*Stefan Felsner, Irina-Mihaela Mustata and Martin Pergel*

The dimension of a partial order $\mathcal{P}$ is the minimum number of linear orders whose intersection is $\mathcal{P}$. There are efficient algorithms to test if a partial order has dimension at most 2. In 1982 Yannakakis showed that for $k \geq 3$ to test if a partial order has dimension $\leq k$ is NP-complete. The height of a partial order $\mathcal{P}$ is the maximum size of a chain in $\mathcal{P}$. Yannakakis also
showed that for \( k \geq 4 \) to test if a partial order of height 2 has dimension \( \leq k \) is NP-complete. The complexity of deciding whether an order of height 2 has dimension 3 was left open. We show that the problem is NP-complete.

Technically we show that the decision problem (3DH2) for dimension is equivalent to deciding for the existence of bipartite triangle containment representations (BTCon). This problem allows a reduction from a class of planar satisfiability problems (P-3-CON-3-SAT(4)) which is known to be NP-hard.

Session 5B

**Flow Diagrams for Trajectory Analysis**

Kevin Buchin, Maike Buchin, Joachim Gudmundsson, Michael Horton and Stef Sijben

We propose movement flow diagrams as a concept to provide a summary for a large number of trajectories and study the problem of computing compact flow diagrams. We show that for a large number of trajectories it is unlikely that efficient algorithms to compute a flow diagram of minimum size exist. More specifically, the problem is W[1]-hard if the number of trajectories is taken as a parameter. For a small number of trajectories we present efficient algorithms.

**Homotopy Measures for Representative Trajectories**

Erin Chambers, Irina Kostitsyna, Maarten Löffler and Frank Staals

An important task in trajectory analysis is defining a meaningful representative for a set of similar trajectories. How to formally define and find such a representative is a challenging problem. We propose and discuss two possible definitions. In both definitions we use only the geometry of the trajectories, that is, no temporal information is required, and measure the quality of the representative using the homotopy area between the representative and the input trajectories. Computing an optimal representative turns out to be NP-hard for one of
the definitions, whereas the other definition allows efficient algorithms for a reasonable class of input trajectories.

**Central Trajectories**

*Marc Van Kreveld, Maarten Löffler and Frank Staals*

We study the problem of computing a suitable representative of a set of similar trajectories. To this end we define a *central trajectory* \( C \), which consists of pieces of the input trajectories, switches from one entity to another only if they are within a small distance of each other, and such that at any time \( t \), the point \( C(t) \) is as central as possible. We measure centrality in terms of the radius of the smallest disk centered at \( C(t) \) enclosing all entities at time \( t \), and discuss how the techniques can be adapted to other measures of centrality. For entities moving in \( \mathbb{R}^1 \) we show that an optimal central trajectory \( C \) representing \( n \) trajectories, each consisting of \( \tau \) edges, has complexity \( \Theta(\tau n^2) \) and can be computed in \( O(\tau n^2 \log n) \) time. For entities moving in \( \mathbb{R}^d \) with \( d \geq 2 \), the complexity of \( C \) is at most \( O(\tau n^{5/2}) \) and can be computed in \( O(\tau n^3) \) time.

**Session 6A**

**Area- and Boundary-Optimal Polygonalization of Planar Point Sets**

*Sándor Fekete, Stephan Friedrichs, Michael Hemmer, Melanie Papenberg, Arne Schmidt and Julian Troegel*

Given a set of points in the plane, we consider problems of finding polygonalizations that use all these points as vertices and that are minimal or maximal with respect to covered area or length of the boundary.

By distinguishing between polygons with and without holes, this results in eight different problems, one of which is the famous Traveling Salesman Problem. Starting from an initial flexible integer programming (IP) formulation, we develop two specific IPs and report preliminary results obtained by our implementation.
Pruning Oracles for High-Dimensional Vector Sets

Stefan Funke and Sabine Storandt

We consider the following problem: Given a set \( S \) of non-negative \( d \)-dimensional cost vectors, we want to compute \( S' \subseteq S \) with \( S' \) containing only vectors \( v \) from \( S \) for which exists an \( \alpha \in \mathbb{R}^d_{\geq 0} \) such that \( \alpha^T v \leq \alpha^T w \) for all \( w \in S \). Based on a geometric interpretation of this problem we propose pruning oracles which reduce \( S \) to \( S' \). We outline how these oracles can be employed in practice to enable personalized route planning in huge street networks. Finally, we prove the effectiveness of the oracles experimentally using a CGAL-based implementation.

Non-crossing Monotonic Paths in Labeled Point Sets on the Plane

Toshinori Sakai and Jorge Urrutia

Let \( n \) be a positive integer, and let \( P \) be a set of \( n \) points in general position on the plane with pairwise different weights. A polygonal line connecting \( k \) points \( p_1, p_2, \ldots, p_k \) of \( P \) is called a monotonic path of length \( k \) if the sequence of the weights of \( p_1, p_2, \ldots, p_k \) is monotonically increasing or decreasing in this order. We show that \( P \) contains a vertex set of a non-crossing monotonic path of length at least \( c(n^{(1/2)}1) \), where \( c=1.0045 \ldots \)

Session 6B

Optimal Straight-line Labels for Island Groups

Arthur van Goethem, Marc Van Kreveld, Andreas Reimer, Maxim Rylov and Bettina Speckmann

Maps are used to solve a wide variety of tasks, ranging from navigation to analysis. Often, the quality of a map is directly related to the quality of its labelling. Consequently, a lot of research has focussed on the automatization of the labelling process. Surprisingly the (automated) labelling of island groups has received little attention so far. This is at least partially caused by the lack of cartographic principles.
Though extensive guidelines for map labelling exist, information on the labelling of groups of islands is surprisingly sparse. We define a formal framework for island labelling. The framework spawns a large series of unexplored computational geometry problems, which are interesting for the CG-community. In this paper we start by looking at a non-overlapping, straight label. We describe two algorithms for a straight-line label that is, or is not, allowed overlap with islands. Furthermore, we discuss several extensions to these algorithms solving closely related problems.

**Clustered Edge Routing**
*Quirijn Bouts and Bettina Speckmann*

The classic method to depict graphs is a node-link diagram where vertices (nodes) are associated with each object and edges (links) connect related objects. However, node-link diagrams quickly appear cluttered and unclear, even for moderately sized graphs. If the positions of the nodes are fixed then suitable link routing is the only option to reduce clutter. We present a novel link clustering and routing algorithm which respects (and if desired refines) user-defined clusters on links. If no clusters are defined a priori we cluster based on geometric criteria, that is, based on a well-separated pair decomposition (WSPD). We route link clusters individually on a sparse visibility spanner. To completely avoid ambiguity we draw each individual link and ensure that clustered links follow the same path in the routing graph. We prove that the clusters induced by the WSPD consist of compatible links according to common similarity measures as formalized by Holten and van Wijk.

**Mosaic Drawings and Cartograms**
*Rafael G. Cano, Kevin Buchin, Thom Castermans, Astrid Pieterse, Willem Sonke and Bettina Speckmann*

Cartograms visualize quantitative data about a set of regions such as countries or states. There are several different types of cartograms
and -- for some -- algorithms to construct them automatically exist. We focus on mosaic cartograms: cartograms that use multiples of simple tiles -- usually squares or hexagons -- to represent regions. Mosaic cartograms communicate data well that consist of, or can be cast into, small integer units (for example, electoral college votes), they allow users to accurately compare regions, and they can often maintain a (schematized) version of the input regions' shapes. We propose the first method to construct mosaic cartograms fully automatically. To do so, we first introduce mosaic drawings of triangulated planar graphs. We then show how to modify mosaic drawings into mosaic cartograms with zero cartographic error while maintaining correct adjacencies between regions.

Session 7A

**Long Paths in Line Arrangements**  
*Udo Hoffmann, Linda Kleist and Tillmann Miltzow*

An arrangement of lines partitions the plane into vertices, edges and faces. A path in a line arrangement is a sequence of faces where every two consecutive faces share an edge and each face occurs at most once. We prove that every arrangement of $n$ lines has a path of length $1/3n^2 - O(n)$. This is tight up to the linear order term.

We also consider arrangements of red and blue lines, where our paths must cross red and blue edges alternatingly:We describe an example of a line arrangement with $3k$ blue and $2k$ red lines with no alternating path longer than $14k$ and show that any arrangement with $n$ lines has a coloring such that there exist an alternating path of length $\Omega(n^2 / \log n)$.

**The Number of Combinatorially Different Convex Hulls of Points in Lines**  
*Heuna Kim, Wolfgang Mulzer and Eunjin Oh*

Given a sequence $R$ of planar lines in general position, we can obtain
a point set by picking exactly on point from each line in \( \mathbb{R} \). We provide exponential upper and lower bounds on the number of combinatorially different convex hulls for point sets that are generated in this manner.

**Distinct distances between points and lines**

*Micha Sharir, Shakhar Smorodinsky, Claudiu Valculescu and Frank de Zeeuw*

We show that for \( m \) points and \( n \) lines in \( \mathbb{R}^2 \), the number of distinct distances between the points and the lines is \( \Omega(m^{1/5}n^{3/5}) \), as long as \( m^{1/2} < n < m^2 \). We also show that for any \( m \) non-collinear points, the number of distances between these points and the lines spanned by them is \( \Omega(m^{4/3}) \).

**Ramsey numbers for empty convex polygons**

*Crevel Bautista-Santiago, Javier Cano, Ruy Fabila-Monroy, Carlos Hidalgo Toscano, Clemens Huemer, Jesús Leaños, Toshinori Sakai and Jorge Urrutia*

We study a geometric Ramsey type problem where the vertices of the complete graph \( K_n \) are placed on a set \( S \) of \( n \) points in general position in the plane, and edges are drawn as straight-line segments.

We define the empty convex polygon Ramsey number \( R_{EC}(k, k) \) as the smallest number \( n \) such that for every set \( S \) of \( n \) points and for every two-coloring of the edges of \( K_n \) drawn on \( S \), at least one color class contains an empty convex \( k \)-gon. A polygon is empty if it contains no points from \( S \) in its interior. We prove \( 17 \leq R_{EC}(3, 3) \leq 463 \) and \( 57 \leq R_{EC}(4, 4) \).

Further, there are three-colorings of the edges of \( K_n \) (drawn on a set \( S \)) without empty monochromatic triangles. A related Ramsey number for islands in point sets is also studied.
Session 7B

Efficient Spanner Construction for Directed Transmission Graphs
Haim Kaplan, Wolfgang Mulzer, Liam Roditty and Paul Seiferth

Let \( P \) be a planar set of \( n \) points, each with an associated radius \( r_p > 0 \). The transmission graph \( G \) for \( P \) has vertex set \( P \) and a directed edge from \( p \) to \( q \) if and only if \( q \) lies in the ball with radius \( r_p \) around \( p \). Let \( t \leq 1 \). A \( t \)-spanner \( H \) for \( G \) is a sparse subgraph such that for any two vertices \( p \) and \( q \) connected by a path of length \( l \) in \( G \), there is a path of length at most \( tl \) from \( p \) to \( q \) in \( H \). Given \( G \) implicitly as points with radii, we show how to compute a \( t \)-spanner for \( G \) in time \( O(n (\log n + \log \Psi)) \), where \( \Psi \) is the ratio of the largest and smallest radius in \( P \).

Constrained Generalized Delaunay Graphs Are Plane Spanners
Prosenjit Bose, Jean-Lou De Carufel and André van Renssen

We look at generalized Delaunay graphs in the constrained setting by introducing line segments which the edges of the graph are not allowed to cross. Given an arbitrary convex shape \( C \), an unconstrained Delaunay graph is constructed by adding an edge between two vertices \( p \) and \( q \) if and only if there exists a homothet of \( C \) with \( p \) and \( q \) on its boundary that does not contain any other vertices. We show that, regardless of the convex shape used to construct the constrained Delaunay graph, there exists a constant \( t \) such that it is a plane \( t \)-spanner.

Two-Level Rectilinear Steiner Trees
Stephan Held and Nicolas Kaemmerling

Given a set \( P \) of terminals in the plane and a partition of \( P \) into \( k \) subsets \( P_1, \ldots, P_k \), a two-level rectilinear Steiner tree consists of a rectilinear Steiner tree \( T_i \) connecting the terminals in each set \( P_i \) (\( i = 1, \ldots, k \)) and a top-level tree \( T_{top} \) connecting the trees \( T_1, \ldots, T_k \). The goal is to minimize the total length of all trees. This problem
arises naturally in the design of low-power physical implementations of parity functions on a computer chip.

For bounded \( k \) we present a polynomial time approximation scheme (PTAS) that is based on Arora's PTAS for rectilinear Steiner trees after lifting each partition into an extra dimension.

For the general case we propose an algorithm that predetermines a connection point for each \( T_i \) and \( T_{top} \) \((i=1,\ldots,k)\). Then, we apply any approximation algorithm for minimum rectilinear Steiner trees in the plane to compute each \( T_i \) and \( T_{top} \) independently.

This gives us a 2.37-factor approximation with a running time of \( O(-P-\log |P|) \) suitable for fast practical computations. The approximation factor reduces to 1.63 by applying Arora's approximation scheme in the plane.

**Max Shortest Path for Imprecise Points**

*Sandro Montanari, Matúš Mihalák and Yann Disser*

Given a set of polygons and an underlying graph connecting them, we consider the problem of placing a point inside each polygon in order to maximize the shortest path distance between two given polygons in the resulting geometric graph. We show that the problem is APX-hard even when the polygons are degenerate and consist only of vertical aligned segments and points. For the special case where the underlying graph is a path, we provide an algorithm computing an optimum placement in time \( O(n \cdot k^2) \), where \( n \) is the number of polygons and \( k \) is the maximum number of corners of a polygon in the set.

**Session 8A**

*Fully autonomous Self-Localization via Trajectory Representations based on Inflection Points*

*Stefan Funke, Robin Schirrmeister, Simon Skilevic and Sabine Storandt*

We present a novel method for self-localization in a fully au-
tonomously manner, in particular not requiring GPS, GSM or WiFi availability. The new method does not rely on distance information but only absolute directional information that can be easily acquired using an electronic compass present in almost every current smartphone. Our approach is based on an inflection-point-based representation of trajectories in the road network and a respective query data structure.

**A Method for Fitting Real World Tessellations with Voronoi Diagrams**

*Supanut Chaidee and Kokichi Sugihara*

There are many phenomena that generate polygonal tessellations on surfaces in 3D space. In this study, we propose a framework for finding the spherical Voronoi diagram that best fits a given photograph of a tessellation on a curved surface. A new distance is introduced in order to define the Voronoi diagram that is suitable for this purpose. Finally, this is reduced to an optimization problem, and numerical results are shown.

**Exact Minkowski Sums of Polygons With Holes**

*Alon Baram, Efi Fogel, Dan Halperin, Michael Hemmer and Sebastian Morr*

We present an efficient algorithm that computes the Minkowski sum of two polygons, which may have holes. The new algorithm is based on the convolution approach. We also observe and prove a property of the Minkowski sum of polygons with holes, which we exploited to expedite the new algorithm.

We introduce a robust implementation of the new algorithm, which follows the Exact Geometric Computation paradigm and thus guarantees exact results. We also present an empirical comparison of the performance of Minkowski sum constructions, where we show that the implementation of the new algorithm exhibits better performance than all other implementations in many cases. In particular, we compared
the implementation of the new algorithm, an implementation of the standard convolution algorithm, and an implementation of the decomposition approach using various convex decomposition methods. The software has been developed as an extension of the 2D Minkowski Sum package of CGAL (Computational Geometry Algorithms Library). Additional information is available at http://acg.cs.tau.ac.il/projects/reduced_convolution/project-page.

Session 8B

**On the Complexity of the Discrete Fréchet Distance under** $L_1$ and $L_\infty$

*Omer Gold and Micha Sharir*

We study the decision tree complexity of the discrete Fréchet distance (decision version) under the $L_1$ and $L_\infty$ metrics over $\mathbb{R}^d$. While algorithms for the Euclidean ($L_2$) discrete Fréchet distance were studied extensively, the problem in other metrics such as $L_1$ and $L_\infty$ seems to be much less investigated.

For the $L_1$ discrete Fréchet distance in $\mathbb{R}^d$ we show a $2d$-linear decision tree with depth $O(n \log n)$, for any constant $d$. For the $L_\infty$ discrete Fréchet distance in $\mathbb{R}^d$ we show a 2-linear decision tree with depth $O(n \log n)$, for any constant $d$. We hope that these near-linear depth decision trees will motivate the study of the problem in these metrics and, in particular, will lead to the development of improved algorithms.

**A Middle Curve Based on Discrete Fréchet Distance**

*Hee-Kap Ahn, Helmut Alt, Maike Buchin, Ludmila Scharf and Carola Wenk*

Given a set of polygonal curves we seek to find a middle curve that represents the set of curves. We ask that the middle curve consists of points of the input curves and that it minimizes the discrete Fréchet distance to the input curves. We develop algorithms for three different variants of this problem.
Computing the Similarity Between Moving Curves
Kevin Buchin, Tim Ophelders and Bettina Speckmann

Over the past years the availability of devices that can be used to track moving objects has increased dramatically, leading to an explosive growth in movement data. Consequently recent years have seen a significant increase in the number of methods developed to extract knowledge from moving object data. Most research efforts have hereby focused on moving point objects. Not all moving objects, however, can be reasonably represented as points. Here we hence want to go beyond this basic setting, by studying moving complex, non-point objects, specifically, moving curves (which can e.g. model changing coastlines, retreating glacier termini, or slithering snakes).

Let \( A \) and \( B \) be two moving curves, both defined by a sequence of polylines. Based on the Fréchet distance, we define various similarity measures between \( A \) and \( B \), and depending on the context from which the curves arise, one similarity measure may be preferable over an other. Since points on a moving curve have two parameters, namely the position along the curve as well as time, \( A \) and \( B \) can be interpreted as surfaces.

While it is known that computing the Fréchet distance between surfaces is NP-hard and it is not even known whether the corresponding decision problem is in NP, we show for variants arising in the context of moving curves that they are polynomial-time solvable or NP-complete depending on the restrictions imposed on how we match \( A \) and \( B \). In particular we show that synchronizing some of the parameters between \( A \) and \( B \) leads to sufficient structure to achieve polynomial time solutions. For these cases we present novel algorithms to compute surfaces through the so-called freespace, which is a three-dimensional space with holes.
Session 9A

Exact solutions for the continuous 1.5D Terrain Guarding Problem
Stephan Friedrichs, Michael Hemmer and Christiane Schmidt

In the NP-hard continuous 1.5-dimensional terrain guarding problem (TGP) we are given an x-monotone chain (the terrain $T$) and ask for the minimum number of point guards (located anywhere on $T$), such that all points of $T$ are covered. We recently gave guard candidate and witness sets of polynomial size, $G$ and $W$, such that there exists a minimum-cardinality guard cover $G^* \subseteq G$ that covers $T$, and when these guards monitor all points in $W$, all of $T$ is guarded. This leads to a PTAS as well as an (exact) IP formulation for TGP.

In this paper, we significantly reduce the size of $G$ and $W$, allowing us to propose an algorithm for reliably finding exact, optimal solutions for instances with 100000 vertices within seconds.

Efficient Algorithms and Implementations for Visibility in 1.5D Terrains
Andreas Haas and Michael Hemmer

Visibility is a well studied problem in computational geometry as it is used as a basic block by many algorithms. In this paper we focus on visibility in 1.5D terrains. We report on new implementations, corresponding experimental evaluations, for an extended version of the sweep line algorithm recently presented by Löffler et al. as well as a variant incorporating ideas of the Triangular Expansion algorithm for polygons of Bungiu et al. Our algorithms are currently submitted as an extension of the upcoming visibility package of the Computational Geometry Algorithms Library (CGAL).

Experiments on Parallel Polygon Triangulation Using Ear Clipping
Günther Eder, Martin Held and Peter Palfrader

We present an experimental study of different strategies for triangulating polygons in parallel. As usual, we call three consecutive vertices of a polygon an ear if the triangle that is spanned by them is com-
pletely inside of the polygon. Extensive tests on thousands of sample polygons indicate that most polygons have a linear number of ears. This experimental result suggests that polygon-triangulation algorithms based on ear clipping might be well-suited for parallelization. We discuss three different approaches to parallelizing ear clipping and report on our experimental findings. Extensive tests show that the most promising method achieves speedups that make it a useful practical enhancement.

Session 9B

Kinetic Conflict-Free Coloring
Mark De Berg, Tim Leijssen and Marcel Roeloffzen
A conflict-free coloring, or CF-coloring for short, of a set $P$ of points in the plane with respect to disks is a coloring of the points of $P$ with the following property: for any disk $D$ containing at least one point of $P$ there is a point $p \in P \cap D$ so that no other point $q \in P \cap D$ has the same color as $p$. In this paper we study the problem of maintaining such a CF-coloring when the points in $P$ move. We present two methods for this and evaluate the maximum number of colors used as well as the number of recolorings, both in theory and experimentally.

Kinetic Data Structures for Clipped Voronoi Computations
Duru Türkoğlu
We consider the mesh refinement problem in the kinetic setting: given an input set of moving points, the objective is to design a kinetic data structure (KDS) for inserting additional so-called Steiner points so that the resulting output set yields a quality triangulation. Therefore, the selection of Steiner points plays a crucial role, both in terms of the output itself and the quality of the triangulation. Although many Steiner point selection schemes have been devised, it is not straightforward how to adapt them in the kinetic setting, regardless of whether it is possible to adapt them or not.
In this paper, we design a KDS by extending a previously proposed query structure which has been employed for Steiner point selection in the dynamic setting. The key geometric property computed in these structures is the clipped Voronoi cells, a locally restricted version of the standard Voronoi cells. Our KDS maintains these clipped Voronoi cells, where each query takes constant time to compute as well as to update. Hence, our KDS is responsive, and it is efficient processing a constant number of events for each query, and it is also local and compact.

Dynamic Convex Hull for Simple Polygonal Chains in Constant Amortized Time per Update
Norbert Bus and Lilian Buzer
We present a new algorithm to construct a dynamic convex hull in the Euclidean plane, supporting insertion and deletion of points. Both operations require amortized constant time. At each step the complete representation of the convex hull is accessible. The algorithm is on-line, does not require prior knowledge of all the points. The only assumptions are that the points have to be located on a simple polygonal chain and that the insertions and deletions must be carried out in the order induced by the polygonal chain.

Session 10A

A local geometry based algorithm for point cloud edge classification
Mihai-Sorin Stupariu
The identification of salient features in point clouds is one of the main application areas of geometric algorithms. In this paper, we deal with classifying high density point clouds on polyhedral objects on the basis of a two-stage classification method. The first step consists in applying Principal Component Analysis, which commonly uses the eigenvalues of the covariance matrix and provides several point classes. In the second stage, a neighbourhood-based analysis is used for refi-
ing the classification. The method relies on the discrete Gaussian curvature, defined as a weighted angular defect and computed for the vertices of a suitably chosen Triangular Irregular Network. A special emphasis is put on identifying jump edges, which correspond to surface discontinuities and crease edges, which occur where two planes of the same object meet. The key issue is that the local geometry of a point cloud is different in a neighbourhood of a jump edge - planar structure, compared to a crease edge, where two planes intersect along the support line of the edge.

*Improved single tree-crown extraction from 3D point clouds*

*Domen Mongus and Borut Žalik*

This paper proposes a new approach for delineation of single tree-crowns in LiDAR data that is based on Locally Fitted Surfaces LoFS. This new definition of watershed markers is, in comparison to the traditionally used local maxima of the smoothed canopy height model, better adapted for dealing with anisotropic shapes of tree-crowns of various sizes. As confirmed by results, approximately 6% increase in overall accuracy is achieved in this way.

*Low-quality dimension reduction and high-dimensional Approximate Nearest Neighbor*

*Evangelos Anagnostopoulos, Ioannis Emiris and Ioannis Psarros*

We generalize the Johnson-Lindenstrauss lemma to define "low-quality" mappings to a Euclidean space of significantly lower dimension which allow us to solve the approximate nearest neighbor problem ($\epsilon$-ANN). With high probability, an approximate nearest neighbor lies among the $k$ approximate nearest neighbors in the projected space. In order to compute them, we employ a data structure, such as BBD-trees.

Our algorithm, given $n$ points in $\mathbb{R}^d$, achieves space usage in $O(dn)$, preprocessing time in $O(dn\log n)$, and query time in $O(d\, n^\rho \log n)$, where $\rho$ is proportional to $1-1/\ln \ln n$, for fixed $\epsilon \in (0, 1)$.  

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The dimension reduction is larger if one assumes that pointsets possess some structure, namely bounded expansion rate. We present experimental results in up to 500 dimensions and $10^5$ points.

**Session 10B**

*Time-Space Trade-offs for Voronoi Diagrams*

Matias Korman, Wolfgang Mulzer, Andre van Renssen, Marcel Roeloffzen, Paul Seiferth and Yannik Stein

Let $S$ be a planar $n$-point set. Classically, one can find the Voronoi diagram $VD(S)$ for $S$ in $O(n \log n)$ time and $O(n)$ space. We study the situation when the available workspace is limited: for $s$ in $\{1, \ldots, n\}$, an $s$-workspace algorithm has read-only access to an input array with the points from $S$ in arbitrary order, and it may use only $O(s)$ additional words of $O(\log n)$ bits for reading and writing intermediate data. We describe a randomized $s$-workspace algorithm for finding $VD(S)$ in expected time $O((n^2/s) \log s + n \log s \log^* s)$. This almost matches the optimal running times for both constant and linear workspace and provides a continuous trade-off between them.

*Linear-Time Algorithms for the Farthest-Segment Voronoi Diagram and Related Tree Structures*

Elena Khramtcova and Evanthia Papadopoulou

We present linear-time algorithms to construct tree-structured Voronoi diagrams with disconnected regions, after the sequence of their faces at infinity or an enclosing boundary is known. We focus on the farthest-segment Voronoi diagram; the approach is also applicable to computing the order-$(k+1)$ subdivision within an order-$k$ segment Voronoi region, and updating the nearest-neighbor diagram after deletion.
β-skeletons for a set of line segments in $\mathbb{R}^2$

Miroslaw Kowaluk and Gabriela Majewska

β-skeletons are well-known neighborhood graphs for a set of points. We extend this notion to sets of line segments and present algorithms computing such skeletons for the entire range of $\beta$ values. The main reason of such extension is a study of β-skeletons for points moving along given line segments. We show that relations between 1-skeleton (Gabriel Graph), 2-skeleton (Relative Neighborhood Graph) and the Delaunay triangulation for sets of points hold also for sets of segments. We present algorithms for computing circle and lune-based β-skeletons. We describe an algorithm that for $\beta \geq 1$ computes the β-skeleton for a set $S$ of $n$ segments in the Euclidean plane in $O(n^2 \alpha(n) \log n)$ time in the circle-based case and in $O(n^2 \lambda_4(n))$ in the lune-based one, where the construction relies on the Delaunay triangulation for $S$. When $0 \leq \beta < 1$, the β-skeleton can be constructed in a $O(n^3 \lambda_4(n))$ time.
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<td>08:45</td>
<td>Official opening and opening remarks, Lecture room P01</td>
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<td>09:00</td>
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<td>Hierarchical Compositional Representations of Structure for Computer</td>
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<td>Hee-Kap Ahn, Luis Barba, Prosenjit Bose, Jean Lou de Carufel, Matias</td>
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<td>Korman and Eunjin Oh: A linear-time algorithm for the geodesic</td>
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<td>center of a simple polygon</td>
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<td>10:50</td>
<td>Leonidas Palios: Computing the s-kernel of Orthogonal Polygons</td>
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<td>10:50</td>
<td>Wolfgang Mulzer and Yannik Stein: Approximating the Colorful</td>
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<td>Caratheodory Theorem</td>
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<td>11:10</td>
<td>Carlos Alegria Galicia, David Orden, Carlos Seara and Jorge Urrutia:</td>
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<td>Optimizing an oriented convex hull with two directions</td>
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<td>Mark de Berg and Ali D. Mehrabi: Finding Pairwise Intersections</td>
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<td>Javad Hasheminejad, Payam Khanteimouri and Ali Mohades: Computing</td>
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<td>the Smallest Color-Spanning Equaliteral Triangle</td>
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<td>Fabian Stehn, Christian Knauer and Luise Sommer: Elastic Shape</td>
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<td>Matching for Translations under the Manhattan Norm</td>
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<td>Hamid Hoorfar and Ali Mohades: Special Guards in Chromatic Art</td>
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<td>Toru Hasunuma: A Linear-Time Algorithm for the Queue-Numbers of Proper Triangulated Cacti</td>
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<td>Boris Albar, Daniel Gonçalves and Kolja Knauer: Orienting triangulations</td>
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<td>Monica Blanco and Francisco Santos: Lattice 3-polytopes with six lattice points</td>
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<td>Colm O Dunlaing: Compact families of Jordan curves and convex hulls in three dimensions</td>
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<td>S. Hoffmann: The Slope Number of Segment Intersection Graphs</td>
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<td>S. Felsner, I-M. Mustata and M. Pergel: The Complexity of the Partial Order Dimension Problem - Closing the Gap</td>
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<td>S. Funke and S. Storandt: Pruning Oracles for High-Dimensional Vector Sets</td>
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<td>and Tillmann Miltzow: Long Paths in Line Arrangements</td>
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<td>Heuna Kim, Wolfgang Mulzer</td>
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<td>and Eunjin Oh: The Number of Combinatorially Different Convex Hulls of Points in Lines</td>
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<td>Micha Sharir, Shakhar</td>
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<td>Crevel Bautista-Santiago,</td>
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<td>Sakai and Jorge Urrutia:</td>
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<td>Conference dinner, Four Points by Sheraton Ljubljana Mons</td>
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<td>Chair: Michael Hemmer</td>
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<td>13:00</td>
<td>Mihai-Sorin Stupariu: A local geometry based algorithm for point cloud edge classification</td>
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<td>Domen Mongus and Borut Žalik: Improved single tree-crow extraction from 3D point clouds</td>
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<td>Evangelos Anagnostopoulos, Ioannis Emiris and Ioannis Psarros: Low-quality dimension reduction and high-dimensional Approximate Nearest Neighbor</td>
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<td>8:45 – 9:00</td>
<td>Opening remarks</td>
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